MASS EXCHANGE BETWEEN SPHERICAL BODIES AND A FLUID STREAM

G. A. Aksel'rud

UDC 532.7

A solution of the problem of determining the mass-exchange coefficient between the surface of a sphere and a liquid flowing over the sphere at low Reynolds number (second approximation) is presented.

In 1952, the author and V. G. Levich independently obtained the following equation [1-3]:

$$\frac{\mathrm{Nu}}{\sqrt[3]{\mathrm{Pr}}} = 0.99\sqrt[3]{\mathrm{Re}}, \tag{1}$$

which is valid for Re < 1. The author used a diffusion integral relation governing the thickness of the diffusion layer:

$$\frac{\partial}{a\partial\theta}\int_0^\delta (c-c_1)\,v_\theta dy + \frac{\mathrm{ctg}\theta}{a}\int_0^\delta (c-c_1)\,v_\theta dy = -D\left(\frac{\partial c}{\partial y}\right)_{y=0}, \tag{2}$$

and the velocity distribution was determined from the known Stokes solution for the stream function

$$\psi = -\frac{U}{4}\sin^2\theta\left(2r^2 - 3ar + \frac{a^3}{r}\right).$$

Recently, Van Dyke [4] obtained a second approximation to describe the fluid flow near a sphere

$$\psi = -\frac{U}{4}\sin^2\theta\,(r-a)^2\left[\left(1+\frac{3}{16}\mathrm{Re}\right)\left(2+\frac{a}{r}\right) + \frac{3}{16}\mathrm{Re}\left(2+\frac{a}{r}+\frac{a^2}{r^2}\right)\cos\theta\right]. \tag{3}$$

The possibility was therefore disclosed for obtaining the second approximation in the solution of the mass-exchange problem.

It follows from (3) that for Re ≤ 16 the tangential velocity near a sphere v_θ is positive on the whole surface of the sphere. If Re > 16, this velocity takes on negative values at the rear part of the sphere for

$$\cos\theta_1 = -\left(\frac{3}{4}+\frac{4}{\mathrm{Re}}\right) = -m. \tag{4}$$

In the first case we can follow the growth of the diffusion layer on the whole surface ($0 \le \theta \le \pi$), on the major portion of the surface ($0 \le \theta \le \theta_1$) in the second case; the conditions of diffusion in the domain $\theta_1 \le \theta \le \pi$ are distinguished by extreme complexity because of the presence of a stationary vortex therein [4]. However, the mass exchange of the part of the surface with $\theta > \theta_1$ can be neglected because this part comprises less than 12.5% of the whole surface of the sphere, and because of the insignificant tangential velocities in this domain.

L'vov Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 1, pp. 110-112, July, 1970. Original article submitted July 28, 1969.

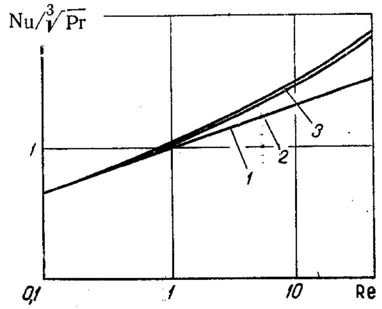

Fig. 1. Critical dependences governing the mass-exchange coefficient: 1) from (1); 2) from (7); (8), 3) from (9).

The solution of the problem under consideration is performed as follows.

1. The tangential velocity v_θ and its value within the diffusion layer, i.e., for $r = a + y$, $y \ll a$, are determined from (3).

2. The concentration distribution in the diffusion layer is represented by the relationship

$$\frac{c - c_1}{c_s - c_1} = 1 - \sin\frac{\pi}{2}\frac{y}{\delta},$$

which satisfies the boundary conditions

$$c(0) = c_s;\ c(\delta) = c_1;\ \left(\frac{\partial c}{\partial y}\right)_{y=\delta} = 0;\ \left(\frac{\partial^2 c}{\partial y^2}\right)_{y=0} = 0.$$

3. Integration in conformity with (2) and subsequent solution of the differential equation determine the diffusion-layer thickness, and its distribution over the surface

$$\delta = \sqrt[3]{x}\frac{a}{\sqrt[3]{\frac{Ua}{D}}}, \tag{5}$$

$$x = \frac{16{,}59\int\limits_0^\theta \sin^2\theta\left[\left(1 + \frac{3}{16}\mathrm{Re}\right) + \frac{\mathrm{Re}}{4}\cos\theta\right]^{1/2} d\theta}{\sin^3\theta\left[\left(1 + \frac{3}{16}\mathrm{Re}\right) + \frac{\mathrm{Re}}{4}\cos\theta\right]^{\frac{3}{2}}}. \tag{6}$$

4. The flux of material from the sphere surface is

$$I = -D\int\limits_0^{\theta_1}\left(\frac{dc}{dy}\right)_{y=0} 2\pi a \sin\theta d\theta$$

and it determines the magnitude of the mass-exchange coefficient

$$I = k(c_s - c_1)\, 2\pi a^2 (1 - \cos\theta_1).$$

The final result is

$$\mathrm{Re} \leqslant 16;\ m \geqslant 1;$$

$$\frac{\mathrm{Nu}}{\sqrt[3]{\mathrm{Pr}}} = 0{,}304\sqrt[3]{\mathrm{Re}\left(1 + \frac{3}{16}\mathrm{Re}\right)}\sqrt[3]{\frac{1+m}{m}}\left[m(1-m)F\left(\sqrt{\frac{2}{1+m}}\right) + (3 + m^2)E\left(\sqrt{\frac{2}{1+m}}\right)\right]^{2/3}; \tag{7}$$

$$\mathrm{Re} \geqslant 16;\ m \leqslant 1;$$

$$\frac{\mathrm{Nu}}{\sqrt[3]{\mathrm{Pr}}} = 0.412\sqrt[3]{\mathrm{Re}\left(1 + \frac{3}{16}\mathrm{Re}\right)}\frac{1}{\sqrt[3]{m}(1+m)}\left[(m-3)(1-m)F\left(\sqrt{\frac{1+m}{2}}\right) + (6 + 2m^2)E\left(\sqrt{\frac{1+m}{2}}\right)\right]^{2/3}. \tag{8}$$

Expanding the elliptic integrals in (7) in series and retaining three terms in each, we obtain result more convenient for use

$$\frac{\mathrm{Nu}}{\sqrt[3]{\mathrm{Pr}}} = 0.99\sqrt[3]{\mathrm{Re}}\,\frac{\left(1+\frac{13}{16}\mathrm{Re}+\frac{17}{128}\mathrm{Re}^2\right)^{2/3}}{1+\frac{7}{16}\mathrm{Re}}. \tag{9}$$

The results (1), (7), (8), (9) are presented in Fig. 1 in logarithmic coordinates. The values determined from (9) exceed the exact value (7) by not more than 5%.

NOTATION

a	sphere radius;
c_1	concentration in the main mass of solution;
c	concentration within the diffusion layer;
c_s	concentration on the sphere surface;
F	complete elliptic integral of the first kind;
k	mass-exchange coefficient;
m	a parameter defined by (4);
U	velocity of fluid motion around the sphere;
v_θ	tangential velocity component;
x	a parameter (see (5) and (6));
y	distance from the sphere surface in a radial direction $0 \leq y \leq \delta$;
D	diffusion coefficient;
r	radius-vector of a point outside the sphere;
I	flux of material from the sphere surface (quantity of material lost by the sphere in unit time);
δ	diffusion-layer thickness;
θ	angular distance from the forward stagnation point;
θ_1	the same at points of vertical-zone formation;
ν	kinematic viscosity;
ψ	stream function;
$\mathrm{Nu} = k \cdot 2a/D$	diffusion Nusselt number;
$\mathrm{Pr} = \nu/D$	diffusion Prandtl number;
$\mathrm{Re} = U \cdot 2a/\nu$	Reynolds number;
E	complete elliptic integral of the second kind.

LITERATURE CITED

1. G. A. Aksel'rud, Zh. Fiz. Khim., 27, 1446 (1953).
2. G. A. Aksel'rud, Theory of Diffusion Extraction of Material from Porous Bodies [in Russian], Izd. LPI, L'vov (1959).
3. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1955).
4. M. Van Dyke, Perturbation Method in Fluid Mechanics, Academic Press, New York (1964).